Available online at www.sciencedirect.com

SCIENCE DIRECT®

Applied Mathematics Letters 18 (2005) 187–190

Applied Mathematics Letters

www.elsevier.com/locate/aml

Numerical solution of an inverse problem of gravimetry for a contact surface

Kh.Kh. Imomnazarov[a,*], P.P. Korovin[b], T.T. Rakhmonov[c]

[a] *Institute of Computational Mathematics and Mathematical Geophysics Siberian Branch, Russian Academy of Sciences, Novosibirsk, Russia*

[b] *Joint Institute for Nuclear Research, Dubna, Moscow Region, Russia*

[c] *Institute of Nuclear Physics, Uzbek Academy of Sciences, Tashkent, Uzbekistan*

Received 1 November 2003; accepted 1 January 2004

Abstract

An inverse problem of gravimetry for a contact surface is considered. Using the critical component method, shapes of a surface are determined numerically at a known depth.

Keywords: Gravitational prospecting; Inverse problem

1. Introduction

Gravitational prospecting is used to study a deep Earth's crust structure, tectonic and petrophysical zoning of large regions, geological mapping of closed territories, search for and prediction of oil and gas deposits, search for and prospecting of solid minerals: coal, ores, and nonmetallics. Gravitational prospecting is applied also to solve some problems of engineering geology, geodesic problems, which study the Earth's figure.

An important problem of interpretation of gravitational anomalies is the problem of determining the parameters of ore bodies using the gravitational values at the Earth's surface. If the density of ore

* Corresponding author.
E-mail address: imom@omzg.sscc.ru (Kh.Kh. Imomnazarov).

doi:10.1016/j.aml.2004.01.005

bodies is known from other sources, the problem consists in determining the surface geometry of an ore body [1].

In this paper, the inverse problem is solved with the use of a critical components method [2,3].

2. Problem statement

Let us introduce the Cartesian system of coordinates, in which the axis z is directed vertically upwards.

A linearized statement of the inverse problem for a contact surface $z(x)$ (the form of the contact surface) infinitely long along the axis y is reduced to the solution of the integral Fredholm equation of the first kind [4,5]:

$$\int_a^b \frac{z(\eta)\mathrm{d}\eta}{(x-\eta)^2+H^2} = f(x). \tag{1}$$

Here H is the deposit depth.

It is known [6,7] that the solution of Eq. (1) is an ill-posed problem. To solve Eq. (1) in [8,9], regularization algorithms are proposed and justified. In [1], an iterative method is proposed.

Using the trapezium method, Eq. (1) is reduced to a system of the linear algebraic equations:

$$A\mathbf{u} = \mathbf{F}. \tag{2}$$

Here $A = (A_{ij})$ is the $m \times n$ dimensionality matrix, $\mathbf{u}$ is an unknown vector of dimensionality n, and $\mathbf{F}$ is the known vector of dimensionality m.

3. Numerical experiments

In accordance with [1], we choose an unknown function of the following form:

$$z(\eta) = \begin{cases} 0, & \text{at } \eta < -1; \\ 1+\eta, & \text{at } -1 \le \eta \le 0; \\ 1-\eta, & \text{at } 0 < \eta < 1; \\ 0, & \text{at } \eta > 1. \end{cases}$$

Substituting this function into (1), we calculate the function $f(x)$ at three different points $x = -0.5, 0, 0.5$ for $H = 0.5$, $a = -1$, and $b = 1$, i.e. we assume in (2) that $m = 3$, and $\mathbf{F} = (f(-0.5),\ f(0),\ f(0.5))^{\mathrm{T}}$, where T is the transposition sign.

Let us set $n = 21$. The exact solution to system of Eq. (2) is a vector of the form

$$\begin{aligned}\tilde{\mathbf{u}} = (&0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9,\\ &1.0, 0.9, 0.8, 0.7, 0.6, 0.5, 0.4, 0.3, 0.2, 0.1, 0.0)^{\mathrm{T}}.\end{aligned}$$

To solve the system of linear algebraic equations (2), we use the critical components method [2]. In this method, the numerical solution of systems $BX = Y$ with ill-posed matrices is reduced to the problem of stable solution of the reduced systems. The ill-posed components of the solutions x_{l_k} are considered separately, and do not participate in the recurrent processes of deriving any components of the solution. Therefore, these components are called critical. The critical component method allows us to find numerically an approximate normal minimum solution, which provides a minimum of the discrepancy

Table 1
Approximate solution to system (2)

ε (%)	u_1	u_2	u_3	u_4	u_5	u_6	u_7	δ (%)
0	0.086	0.203	0.242	0.292	0.359	0.448	0.560	0.091
5	0.091	0.213	0.254	0.307	0.377	0.470	0.588	0.104
10	0.095	0.253	0.266	0.321	0.395	0.493	0.616	0.135
15	0.099	0.234	0.278	0.336	0.413	0.515	0.644	0.175
20	0.104	0.244	0.290	0.351	0.431	0.537	0.672	0.219
ε (%)	u_8	u_9	u_{10}	u_{11}	u_{12}	u_{13}	u_{14}	δ (%)
0	0.691	0.824	0.928	0.968	0.928	0.824	0.691	0.091
5	0.725	0.865	0.975	1.016	0.975	0.865	0.725	0.104
10	0.760	0.906	1.020	1.065	1.021	0.906	0.760	0.135
15	0.795	0.947	1.067	1.114	1.067	0.947	0.795	0.175
20	0.829	0.989	1.114	1.162	1.113	0.989	0.829	0.219
ε (%)	u_{15}	u_{16}	u_{17}	u_{18}	u_{19}	u_{20}	u_{21}	δ (%)
0	0.560	0.448	0.359	0.292	0.242	0.203	0.086	0.091
5	0.588	0.470	0.377	0.307	0.254	0.213	0.091	0.104
10	0.616	0.493	0.395	0.321	0.266	0.223	0.095	0.135
15	0.644	0.515	0.413	0.336	0.278	0.234	0.099	0.175
20	0.672	0.537	0.431	0.351	0.290	0.244	0.104	0.219

norm:

$$(X^+ = B^+Y) : \|BX^+ - Y\| = \inf_{X \in X_B} \|BX - Y\|, \|X^+\| = \inf_{X \in X_B} \|X\|,$$

where X_B is the totality of all solutions of the system $BX = Y$, and the unique matrix B^+ satisfies the conditions

$$\|B^+B - E\| = \inf_{\tilde{B}^{-1} \in \Omega_B} \|\tilde{B}^{-1}B - E\|, \|B^+\| = \inf_{\tilde{B}^{-1} \in \Omega_B} \|\tilde{B}^{-1}\|, B^+B = BB^+.$$

Here E is a unit matrix and Ω_B is the totality of all $\tilde{B}^{-1}$, which are "pseudoinverse" to B.

In Table 1, approximate solutions to the system of equations (2) are represented. The relative solution errors $\delta = \|\mathbf{u} - \tilde{\mathbf{u}}\|/\|\tilde{\mathbf{u}}\|$ at the corresponding relative perturbations of the right-hand sides of $\varepsilon = \|\mathbf{F} - \tilde{\mathbf{F}}\|/\|\tilde{\mathbf{F}}\|$ are also given.

References

[1] I.V. Boikov, N.V. Moiko, An iterative method for solving the inverse Gravity problem, Fizika Zemli. (2) (1999) 52–56 (in Russian).

[2] G.A. Emel'yanenko, M.G. Emelianenko, T.T. Rakhmonov, E.B. Dushanov, G.Yu. Konovalova, On efficiency of critical-component method for solving degenerate and ill-posed systems of linear algebraic equations, JINR, Preprint, E11-98-302, Dubna, 1998, p. 36.

[3] G.A. Emel'yanenko, E.B. Dushanov, M.G. Emel'yanenko, T.T. Rakhmonov, A.P. Sapozhnikov, JINRLINPARK—the package of computer-independent programmes for solving ill-posed systems of linear algebraic equations, JINR, R11-2000-287, Dubna, 2000, p. 47 (in Russian).

[4] E.A. Mudretsova (Ed.), Gravitational Prospecting, Nauka, Moscow, 1981, p. 698 (in Russian).

[5] V.N. Strackov, Some quations of the two-dimensional problem of gravimetry, Izv. Fizika Zemli (12) (1970) 32–44 (in Russian).

[6] A.N. Tikhonov, V.Y. Arsenin, Solution of Ill-Posed Problems, Winston, New York, 1977.
[7] M.M. Lavrent'ev, V.G. Romanov, S.P. Shishat.skiĭ, Ill-posed Problems of Mathematical Physics and Analysis, Providence, Rhode Island, 1986.
[8] A.N. Tikhonov, V.B. Glasko, On an approximate solution of integral Fredholm equations of the first kind, Zh. Vychisl. Mat. Mat. Fiz. 4 (3) (1964) 564–571 (in Russian).
[9] V.G. Filatov, Application of method Tsuboi in inverse problems for gravitational prospecting, Prikladnaya Geofizika (68) (1972) 147–152 (in Russian).